INDIAN SCHOOL AL WADI AL KABIR

Practice Paper (2022-2023)-Assessment -I

Class: XII Sub: MATHEMATICS (041) Max Marks: 80

Date: 01-09-2022 Time: 3 hours

General Instructions:

1. This question paper contains six sections- A, B, C, D, E and F. Each part is compulsory.
2. Section – A has 16 objective type questions of 1 mark each.
3. Section – B has 8 MCQ type questions of 1 mark each.
4. Section – C has 2 Case based questions.
5. Section - D has 8 short answer type (SA1) questions of 2 marks each.
6. Section – E has 4 short answer type (SA2) questions of 3 marks each.
7. Section - F has 4 long answer type questions (LA) of 5 marks each.
8. There is an internal choice in some of the questions.

SECTION – A

1	If n(A) = 3, then how many reflexive relations are possible in set A?	1
2	A relation R in S= {4, 2, 3} is defined as R= {(4, 4), (4, 2), (2, 2), (3, 3)}. Find the element of the relation R to be removed to make it as an equivalence relation	1
3	Let A = {1, 2, 3, 4}. Let R be the equivalence relation on A × A defined by (a, b) R (c, d) if a + d = b + c. Then find the equivalence class [(1, 3)] is OR Find the maximum number of equivalence relations on the set A = {2, 3, 4}	1
4	State the reason why the relation R = $\{(a,b): a \leq b^2\}$ on the set R of real numbers is not reflexive.	1
5	A relation R in the set of real numbers R defined as R = $\{(a,b): \sqrt{a} = b\}$ is a function or not. Justify	1
6	Let f: [2, ∞) → R be the function defined by f(x) = $x^2 - 4x + 5$, then find the range of f	1
7	$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$ is equal to	1
8	Write the principal value of $\tan^{-1}(\sqrt{3}) + \cot^{-1}(-\sqrt{3})$ OR Find the value of $2\sec^{-1}2 + \sin^{-1}(\frac{1}{2})$	1
9	Evaluate $\int \frac{x^3}{x+1}\,dx$	1

10	Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+sin\theta & 1 \\ 1 & 1 & 1+cos\theta \end{vmatrix}$	1
11	if $\begin{vmatrix} x & sin\theta & cos\theta \\ -sin\theta & -x & 1 \\ cos\theta & 1 & x \end{vmatrix}$ = 8, then find the value of x	1
12	If A is a square matrix of order 3 such that |adj A| = 64, then |A| is OR If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to	1
13	Evaluate $\int \frac{sinx+cosx}{\sqrt{1+\sin 2x}}\, dx$	1
14	Find the value of k (k < 0) for which the function *f* defined as f(x) = $\begin{cases} \frac{1-coskx}{x\, sinx}, & x \neq 0 \\ \frac{1}{2}, & ,x = 0 \end{cases}$ is continuous at x = 0	1
15	If $e^x + e^y = e^{x+y}$, then find $\frac{dy}{dx}$ OR If x = a secθ, y = b tanθ , then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$	1
16	Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is strictly increasing:	1

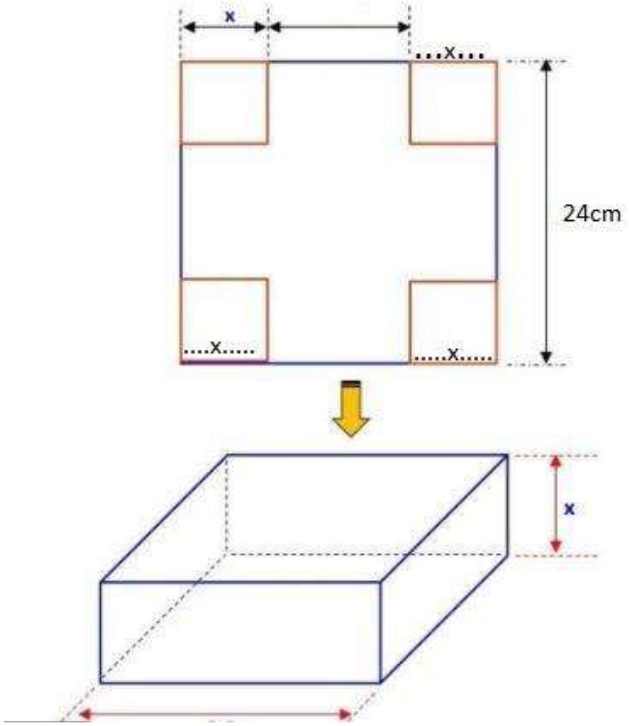
SECTION – B

17	The set of points where the functions f given by f(x) = $\|x - 3\|cosx$ differentiable is (a) R (b) R -{3} (c) (0, ∞) (d) none of these	1
18	A = $\begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$, then the value of x satisfying $0 < x < \frac{\pi}{2}$ when $A + A^T = \sqrt{2}\, I_2$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{5}$	1
19	If a function f: A x B → B x A is defined by f (a, b) = (b, a) on two sets A and B, then the function is (a) Many-one (b) One-one but not onto (c) One-one and onto (d) Neither one-one nor onto	1
20	In a linear programming problem, the constraints on the decision variables x and y are x – 3y ≥ 0, y ≥ 0 *and* 0 ≤ x ≤ 3, then the feasible region (a) is not in the first quadrant (b) is bounded in the first quadrant (c) is unbounded in the first quadrant (d) does not exist	1
21	If y = $\sqrt{sinx + \sqrt{sin\, x + \sqrt{sin\, x + \ldots\ldots\infty}}}$, then $(2y - 1)\frac{dy}{dx}$ is equal to (a) sin x (b) cos x (c) – cos x (d) -sin x	1

22	If A = $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$, then A^{100} is (a) $2^{100}A$ (b) $2^{98}A$ (c) $2^{99}A$ (d) $2^{97}A$	1
23	The interval on which the function f (x) = $2x^3 + 9x^2 + 12x - 1$ is decreasing is (a) (-1, ∞) (b) (-2, -1) (c) (-∞, -2) (d) (-1, 1)	1
24	If y = tan (x +y), then $\frac{dy}{dx}$ is equal to (a) -Cosec^2(x + y) (b) Cosec^2(x + y) (c) -Cosec(x + y) (d) Cosec(x + y)	1

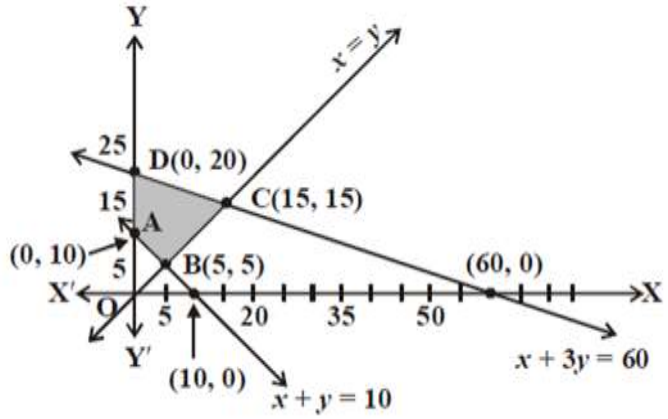
SECTION - C

25	**CASE-BASED/DATA-BASED** Two schools A and B want to award their selected students on the values of Honesty, Hard work, and Punctuality. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to its 3, 2 and 1 students respectively with a total award money of Rs.2200. School B wants to spend Rs. 3100 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as school A). If the total amount of award for one prize on each value is Rs. 1200. Using the concept of matrix and determinants answer the following questions. 	
i)	What is the award money for Honesty? (a) □ 350 (b) □ 300 (c) □ 500 (d) □ 400	1
ii)	What is the award money for Punctuality? (a) □ 300 (b) □ 280 (c) □ 450 (d) □ 500	1
iii)	What is the award money for Hard work? (a) □ 500 (b) □ 400 (c) □ 300 (d) □ 550	1
iv)	If P and Q are two matrix such that PQ = Q and QP = P, then $\lvert Q^2 \rvert$ is equal to (a) $\lvert Q \rvert$ (b) $\lvert P \rvert$ (c) 1 (d) 0	1
26	A square piece of tin 24cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box.	

	 Based on the above information answer the following:	
i)	What is the length of side of the square to be cut from each corner so that the box will have maximum volume?	2
ii)	What is the inner surface area of the box with maximum volume?	2

SECTION – D (Each question carries 2 marks)

27	Show that the relation R in the set Z of integers given by R = {(a, b): 3 divides a – b} is an equivalence relation.	2
28	Express $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$, where $\frac{-\pi}{4} < x < \frac{\pi}{4}$ in the simplest form	2
29	If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}	2
30	Find $\int \frac{1}{cos^2x(1-tanx)^2}\,dx$	2
31	The sides of an equilateral triangle are increasing at the rate of 2cm/sec. Find the rate at which the area increases, when side is 10 cm	2
32	Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function Z = 3x + 9y maximum?	2

		
33	Show that the relation R on a set of real numbers defined by *xRy* if x - y + $\sqrt{2}$ is irrational is reflexive but neither symmetric nor transitive	2
34	If y = $\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then find $\frac{dy}{dx}$	2

SECTION – E (Each question carries 3 marks)

35	Find X so that X $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$	3
36	Find the equation of the line joining A (1, 3) and B (0, 0) using determinants and find k if D (k, 0) is a point such that area of triangle ABD is 3sq units.	3
37	If y = $e^{xsin^2x} + (\sin x)^x$, find $\frac{dy}{dx}$ OR If x = a sin2t (1 +cos2t) and y = b cos2t (1 – cos2t), find $\frac{dy}{dx}$ at t = $\frac{\pi}{4}$	3
38	Evaluate $\int \frac{1}{1+cotx}\,dx$ OR Evaluate $\int \frac{1}{1+tanx}\,dx$	3

SECTION – F (Each question carries 5 marks)

39	Use product $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ to solve the system of equation x + 3z = 9, -x + 2y -2z = 12 and 2x -3y + 4z = 3 OR	5

	If A= $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, then find A^{-1}. Using A^{-1} solve the set of equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$.	
40	If $y = \frac{x \cos^{-1} x}{\sqrt{1-x^2}} - \log\sqrt{1 - x^2}$, then prove that $\frac{dy}{dx} = \frac{\cos^{-1} x}{(1-x^2)^{3/2}}$	5
41	Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.	5
42	A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week? OR Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture	5